

New Model-independent Method to Test the Curvature of Universe

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ABSTRACT

We propose a new model-independent method to test the cosmic curvature by comparing the proper distance and transverse comoving distance. Using the measurements of Hubble parameter $H(z)$ and angular diameter distance d_A , the cosmic curvature parameter Ω_K is constrained to be -0.09 ± 0.19 , which is consistent with a flat universe. We also use Monte Carlo simulation to test the validity and efficiency, and find that our method can give a reliable and efficient constraint on cosmic curvature. Compared with other model-independent methods testing the cosmic curvature, our method can avoid some drawbacks and give a better constraint.

Subject headings: cosmological parameters - cosmology: observations

1. Introduction

The cosmic curvature is a fundamental parameter for cosmology. Whether the space of our Universe is open, flat, or closed is important for us to understand the evolution of our universe, and dark energy equation of state (Clarkson, Cortès, & Bassett 2007; Zhao et al. 2007; Ichikawa et al. 2006; Weinberg et al. 2013). Due to the strong degeneracy between the curvature and the dark energy equation of state, it is difficult to study a non-flat ω CDM model (Clarkson, Cortès, & Bassett 2007). Besides, a significant detection of a non-zero curvature will affect the fundamental theory of cosmology because most observations support a flat Λ CDM model, including the latest Planck result which gives $|\Omega_k| < 0.005$ (Planck Collaboration et al. 2015). However, most of these constraints are not in a direct geometric way. Therefore, determining the cosmic curvature with model-independent methods is very important.

In order to constrain the cosmic curvature in a direct geometric way, some definitions of cosmological distance should be introduced. Several distance definitions, such as the proper distance d_P , luminosity distance d_L , angular diameter distance d_A and transverse comoving distance d_M are defined to investigate cosmology (Hogg 1999; Coles & Lucchin 2002; Weinberg 2008; Weinberg et al. 2013). Under the assumption of Friedmann-Lemaître-Robertson-Walker metric, the proper distance can be expressed as

$$d_P(r) = a_0 \int_0^r \frac{dr'}{\sqrt{1 - Kr'^2}} = a_0 f(r), \quad (1)$$

with $f(r) = \sin^{-1} r$, r , or $\sinh^{-1} r$ for curvature $K = +1$, 0 , or -1 , a_0 is the present scale factor, and r is the comoving coordinate of the source. With the definition of Hubble parameter $H(z) = \dot{a}/a$, it can also be expressed as

$$d_P(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}, \quad (2)$$

where z is the redshift, H_0 is the Hubble constant, c is the speed of light and $E(z) = H(z)/H_0$. Similarly, the transverse comoving distance can be expressed as

$$d_M(z) = a_0 r(z) = \frac{c}{H_0 \sqrt{-\Omega_K}} \sin[\sqrt{-\Omega_K} \int_0^z \frac{dz'}{E(z')}], \quad (3)$$

where Ω_K is the energy density of cosmic curvature ($-i \sin(ix) = \sinh(x)$ if $\Omega_K > 0$). With the definition of d_M , d_L and d_A can be derived through $d_L = d_M(1+z)$ and $d_A = d_M/(1+z)$, respectively.

Numerous works have been done to determine the curvature parameter Ω_K using different methods, some of which are model-independent. Bernstein (2006) proposed a model-independent method using the weak lensing and baryon acoustic oscillation (BAO) data to constrain Ω_K based on the distance sum rule, which was used to test the FLRW metric in Räsänen et al. (2015) (hereafter, called DSR method). The basic principle of DSR method is that the relation between $d(z_s)$ and $d(z_l) + d(z_l, z_s)$ depends on the cosmic geometry (see Fig. 1 of Bernstein (2006)). The value of $d(z_l, z_s)$ can be calculated from gravitational lensing. However, the large uncertainty in gravitational lens system restricts its efficiency on constraining the curvature (Räsänen et al. 2015). Another important model-independent method was proposed in Clarkson, Cortês, & Bassett (2007), by comparing the Hubble parameter $H(z)$ and the derivative function of transverse comoving distance d_M gained from d_A , which has been used in many works (Clarkson et al. 2008; Yahya et al. 2014; Li et al. 2014; Cai et al. 2016) (hereafter, C07 method). The basis of this method is that one can determine the curvature by combining measurements of the Hubble parameter $H(z)$ and the

transverse comoving distance $d_M(z)$

$$\Omega_K = \frac{[H(z)d'_M(z)]^2 - c^2}{[H_0 d_M(z)]^2}, \quad (4)$$

where $'$ means the derivative with respect to redshift z . However, in this method, one needs to determine the derivative function of transverse comoving distance d_M from a fitting function, which will introduce a large uncertainty.

Therefore, in order to avoid the drawbacks of the two methods, we propose a new direct geometric method to test the cosmic curvature. This method is based on the comparison between proper distance d_P obtained from Hubble parameter measurement, and transverse comoving distance d_M obtained from angular diameter distance d_A measurement. The structure of this paper is organized as follows. In section 2, we introduce our new model-independent method to test the cosmic curvature using d_P and d_M . In section 3, we give our constraint on Ω_K using the Hubble parameter and angular diameter distance measurements. In section 4, we test the validity and efficiency of our method with Monte Carlo simulation. In section 5, we discuss its advantages compared with other methods. Finally a summary will be given in section 6.

2. Method to test Ω_K

Comparing the definitions of proper distance d_P and transverse comoving distance d_M , one can find that the difference between them only caused by the curvature of universe. This gives the basis to test the cosmic curvature using the comparison of d_P and d_M . From equations (2) and (3), Ω_K can be derived from

$$\frac{H_0 d_M}{c} \sqrt{-\Omega_K} = \sin\left(\frac{H_0 d_P}{c} \sqrt{-\Omega_K}\right). \quad (5)$$

Equation (5) gives the direct relation among Ω_K , d_P and d_M . Once the d_P and d_M are determined, Ω_K can be calculated through this equation. If the value of redshift and Ω_K are not large, $H_0 d_P \sqrt{-\Omega_K}/c$ is less than one. From the Taylor expansion, the equation (5) can be approximated as

$$\Omega_K = \frac{6c^2}{H_0^2} \frac{d_M - d_P}{d_P^3}, \quad (6)$$

from which Ω_K can be determined directly.

Figure 1 shows the key principle of our method in the $\Omega_K < 0$ case. In this figure, the arc OS is the proper distance between source S and observer O , while the transverse

comoving distance between them is $d_M = a_0 \sin(\frac{d_P}{a_0})$. In this case, it is obvious that the d_M of an object has an up limit a_0 and it is less than d_P . In contrast, in an open universe ($\Omega_K > 0$), we have $d_M > d_P$. $d_M = d_P$ only happens in a flat universe. Therefore, the cosmic curvature can be derived by comparing d_M and d_P . Under the assumption of $\sqrt{|\Omega_K|}I \ll 1$, we can obtain

$$\delta \equiv \frac{d_P - d_M}{d_P} = -\frac{\Omega_K I^2}{6}, \quad (7)$$

with $I = \int_0^z \frac{dz'}{E(z')}$. The δ means the relative difference between d_P and d_M . The value of $|\delta|$ gives the requirement on the accuracy of measurement, which means the Ω_K cannot be constrained if the observed uncertainty is much larger than $|\delta|$. Besides, if the total relative error of the measurement sample of d_M and d_P is σ , one can expect that the tightest constrain on Ω_K will have an error about $\sigma_{\Omega_K} \sim 6\sigma/I^2$. In other words, equation (7) gives the constraint limit of this method.

To obtain the transverse distance d_M , we choose the angular diameter distance d_A measurement based on BAO in several previous works (Blake et al. 2012; Xu et al. 2013; Samushia et al. 2014; Delubac et al. 2015). These data and their references are listed in Table 1. The detailed information about these data can be found in their references. The d_M can be easily derived with the direct relation between them $d_M = d_A(1+z)$. The next important issue is how to measure d_P . From equation (2), the proper distance d_P only depends on the $H(z)$ function. Therefore, in order to derive the proper distance d_P , one can construct the $H(z)$ function from Hubble parameter measurements. Then d_P can be derived from equation (2). There are tens of Hubble parameter measurements derived from differential ages of galaxies and the radial BAO in the previous literature, which are listed in Table 2. In order to make our method model-independent, Gaussian Process (GP) method is used to reconstruct the $H(z)$ function. GP method is a powerful tool to reconstruct a function from data directly without any assumption of the function form and is used widely in astronomy (Holsclaw et al. 2010; Shafieloo & Clarkson 2010; Shafieloo et al. 2012; Seikel et al. 2012a; Bilicki & Seikel 2012). Therefore, with GP method, we don't need any prior cosmological model. There is a good python package for GP method called Gapp developed by Seikel et al. (2012a) which was used in many works (Seikel et al. 2012b; Bilicki & Seikel 2012; Cai et al. 2016). It can reconstruct the function as long as observed data was input. More detailed information about GP method and Gapp can be found in Seikel et al. (2012a).

The main route of our method is that: I) deriving the transverse comoving distance d_M from angular diameter distance d_A measurements; II) reconstructing $H(z)$ function from Hubble parameter measurements using GP method; III) using equation (2) to calculate the proper distance at a certain redshift; IV) using equation (5) or (6) to determine Ω_K at a certain redshift. Several Ω_K can be determined at different redshifts since there are several

d_A measurements. One can choose the average of them as the final result through

$$\Omega_K = \frac{\sum_i \Omega_{K,i} / \sigma_{\Omega_{K,i}}^2}{\sum_i 1 / \sigma_{\Omega_{K,i}}^2} \quad (8)$$

where $\Omega_{K,i}$ and $\sigma_{\Omega_{K,i}}$ are determined Ω_K and its uncertainty at a certain redshift. The total uncertainty can be obtained from

$$\sigma_{\Omega_K}^2 = \frac{1}{\sum_i 1 / \sigma_{\Omega_{K,i}}^2}. \quad (9)$$

3. Results

We collect 31 Hubble parameter measurements from previous literature and list them in table 2. These Hubble parameters at different redshifts are derived using differential ages of galaxies and the radial BAO method. Using these observed data and GP method, the $H(z)$ function can be reconstructed, which is shown as the blue curve in Figure 2. Hereafter, this $H(z)$ function is called GP- $H(z)$. For comparison, we also fit the observed data based on Λ CDM model which is shown as the green curve in Figure 2. Hereafter, this $H(z)$ function is called Fit- $H(z)$. We can find that the Fit- $H(z)$ is well covered by the GP- $H(z)$ function and its 1σ confidence region. With the reconstructed GP- $H(z)$ function, one can use equation (2) to derive the proper distance at a certain redshift. The derived $d_P(z)$ functions from GP- $H(z)$ and Fit- $H(z)$ are shown in Figure 3. Hereafter, they are called GP- $d_P(z)$ and Fit- $d_P(z)$ respectively. Comparing the GP- $H(z)$ and Fit- $H(z)$, one can find that the derivation between them becomes smaller than that between GP- $H(z)$ and Fit- $H(z)$.

Using equation (5), one can derive the Ω_K from the d_P and d_M at the same redshift. Figure 4 shows the result of derived Ω_K at different redshifts. The average Ω_K and its error bar are derived from equations (8) and (9). The result is listed in Table 3. The average Ω_K constrained by these six d_A data is $\Omega_K = -0.09 \pm 0.19$. There is no significant deviation from a flat universe. From bottom panel of Figure 2, it is obvious that the high-redshift measurement gives tighter constraint on Ω_K than the low-redshift measurement. The reason is that the I term in equation (7) is larger at high redshift, which will decrease the error of Ω_K through $\sigma_{\Omega_K} \sim 6\sigma/I^2$.

4. Simulation

In order to test the validity and efficiency of our method, we perform Monte Carlo simulation. The route of simulation is as follows: I) creating mock $H(z) - z$ and $d_M - z$ data sets based on a prior cosmological model; II) reconstructing the GP- $H(z)$ function from GP method; III) using GP- $H(z)$ function to derive the GP- $d_P(z)$ function through equation (2); IV) using equations (5) and (8) to constrain Ω_K and its average value; V) simulating 10^4 times for each prior cosmological model and give the distribution of determined average Ω_K .

For simulation, we choose Λ CDM model as the prior cosmological model. The model parameters are chosen as $H_0 = 70$ km/s/Mpc, $\Omega_M = 0.3$ and $\Omega_\Lambda = 1 - \Omega_M - \Omega_K$ where $\Omega_K = -0.1, 0$ and 0.1 for different cases. In each simulation, there are 20 mock $H - z$ and $d_M - z$ data sets respectively. The redshifts of these mock data are chosen equally in $\log(1+z)$ space in redshift range $0.1 \leq z \leq 5.0$. The relative uncertainty of these mock data is 1% which will be realized in future observation (Weinberg et al. 2013).

Figure 5 gives an example of the simulations in the $\Omega_K = 0$ case. The three panels show the mock Hubble parameter data with GP- $H(z)$ function, mock d_M data with GP- $d_P(z)$ function and the final determined Ω_K . From this figure, we can see that GP method can reconstruct $H(z)$ function well. In this case, the final derived average Ω_K is $\Omega_K = 0.0001 \pm 0.0092$. Figure 6 shows the posterior distributions of Ω_K for three Ω_K cases. From this figure, one can find that our method can give a reliable and tight constraint on the prior Ω_K . The uncertainty is $\sigma_{\Omega_K} \approx 0.011$. This result means that if there are 20 d_A and $H(z)$ measurements with 1% uncertainty, our method can give a constraint on Ω_K at 1% level. In future, there will be more accurate measurements of d_A and $H(z)$, tighter constraint on Ω_K can be expected.

5. Compared with other methods

In this section, we compare our method with other model-independent methods. Just as introduced in the first section, there are two model-independent methods to constrain the curvature of universe proposed in previous literature (Bernstein 2006; Clarkson, Cortés, & Bassett 2007), which have been used in many works (Clarkson et al. 2008; Yahya et al. 2014; Li et al. 2014; Räsänen et al. 2015; Cai et al. 2016). The first one is DSR method based on the distance sum rule proposed by Bernstein (2006) and the other is C07 method based on the equation (4) (Clarkson, Cortés, & Bassett 2007).

The basic principle of DSR method is the distance sum rule which means that if there are two sources S_1 and S_2 at redshift z_1 and z_2 (assuming $z_1 < z_2$), it has $d(z_2) = d(z_1) + d(z_1, z_2)$

if our universe is flat. Otherwise, $d(z_2) > d(z_1) + d(z_1, z_2)$ or $d(z_2) < d(z_1) + d(z_1, z_2)$ for $\Omega_K > 0$ or $\Omega_K < 0$ respectively. The distance between S_1 and S_2 can be determined by gravitational lens (Bernstein 2006; Räsänen et al. 2015). Compared the Figure 1 of Bernstein (2006) with our Figure 1, it can be found that the difference between d_P and d_M is larger than that between $d(z_1) + d(z_1, z_2)$ and $d(z_2)$, which means our method is more sensitive on the cosmic curvature. An accurate calculation gives the relative difference between $d(z_1) + d(z_1, z_2)$ and $d(z_2)$ is

$$|\delta_{DSR}| \leq \left| \frac{\Omega_K I^2}{8} \right|, \quad (10)$$

where $=$ is only valid when the source S_1 locates at the middle of S_2 and observer. Compared with equation (7), one can find that DSR method needs a higher accuracy of measurement than our method. Besides, the systematic uncertainty of gravitation lens system parameter f^2 is about 20% (Kochanek et al. 2000), where f is a phenomenological coefficient that parameterizes uncertainty due to difference between the velocity dispersion of the observed stars and the underlying dark matter, and other systematic effects in strong lensing. This systematic uncertainty is hardly to remove, which restricts its efficiency on constraining the cosmic curvature.

For the C07 method, it is based on equation (4), which needs the first derivative function of $d_M(z)$ fitted from observational data. As is known to all that calculating the derivative function from a fitted function will increase the uncertainty significantly especially when the function form is unknown and the data is not enough. In order to check the efficiency of C07 method, we also use Monte Carlo simulation to test it, and the simulation is same as introduced in section 4. Figure 7 gives an example of the simulations in the $\Omega_K = 0$ case, which is similar with Figure 5. Instead of $H(z)$ and $d_P(z)$ functions, we show the $d'_M(z)$ function in the top panel of Figure 7. Because the $H(z)$ and $d_P(z)$ functions are similar with those in Figure 5. From Figure 7, it is obvious that the $d'_M(z)$ function derived from GP method has a large derivation with the theoretical one, and the determined Ω_K are not as well as those in bottom panel of Figure 5. Figure 8 shows the posterior distributions of Ω_K determined with C07 method for three Ω_K cases. From this figure, one can find that C07 method gives a large uncertainty on the determined Ω_K .

6. Summary

We have proposed a new model-independent method to test the cosmic curvature in this paper. The main principle of our method is to compare the proper distance d_P and transverse comoving distance d_M at same redshift (Figure 1 gives an illustration). Using

equation (5), one can derive the Ω_K if d_P and d_M are obtained. With the measurements of Hubble parameter, we use GP method to reconstruct the $H(z)$ function and use equation (2) to derive the $d_P(z)$ function. Using the measurements of angular diameter distance, transverse comoving distance can be calculated easily through $d_M = d_A(1+z)$. We used the $H(z)$ and d_A measurements collected from previous literature. The reconstructed $H(z)$ function and the derived $d_P(z)$ function are shown in Figures 2 and 3. In order to compare with Λ CDM model, the best-fitted $H(z)$ and $d_P(z)$ function are also shown in these figures. The comparison shows that the GP method can give a reliable reconstructed function from observed data. Figure 4 shows the derived Ω_K at several different redshifts, which are also listed in Table 3. Using equations (8) and (9), the average Ω_K can be obtained, which is $\Omega_K = -0.09 \pm 0.19$. This result shows that Ω_K has no significant derivation from non-zero value.

To check the validity and efficiency of our method, we use Monte Carlo simulation to test it. For Λ CDM model with three different Ω_K , -0.1 , 0 and 0.1 , we simulate 10^4 times for each case. Figure 5 gives an example of the simulations for $\Omega_K = 0$ case and Figure 6 gives the posterior distributions of Ω_K determined with our method for the three Ω_K cases. These two figures show that our method can give a reliable and efficient constraint on Ω_K . We also compared our method with the DSR and C07 method. We find that DSR method needs a higher accuracy of measurement than our method. More importantly, the systematic uncertainty of gravitational lens system parameter f^2 restricts significantly its efficiency on constraining the curvature. For the C07 method, we also test it with simulations. The result is shown in Figures 7 and 8. From Figure 7, we find that the first derivative function of $d_M(z)$ derived from GP method has a large derivation with the theoretical one. So the determined Ω_K not reliable. Figure 8 shows the posterior distributions of Ω_K determined by C07 method for the three Ω_K cases. Meanwhile, the C07 method will give a large uncertainty on the determined Ω_K with observed data at same accuracy level.

Future observations will improve the constraint on the cosmic curvature. The Extended Baryon Oscillation Spectroscopic Survey (eBOSS) will compile 250,000 new, spectroscopically confirmed luminous red galaxies, which yield measurements of d_A with 1.2% precision and measurements of $H(z)$ with 2.1% precision (Dawson et al. 2016). HETDEX will perform a survey of 800,000 Ly α emission-line galaxies at $1.8 < z < 3.7$ (Hill et al. 2006). The precision on d_A and $H(z)$ is of order 2% using BAO. The BAO analysis from Wide Field Infrared Survey Telescope (WFIRST) will yield about 1.0% measurements of the angular diameter distance d_A and Hubble parameter $H(z)$ by 17 million galaxies redshift survey in the redshift range $1.3 < z < 2.7$ (Green et al. 2012). Meanwhile, the Euclid satellite with survey area of approximately 14,000 deg² and redshift range $0.7 < z < 2.0$ will measure Hubble parameter $H(z)$ with 1.5% precision (Laureijs et al. 2011). Weinberg et al. (2013) had predicted the

accuracy of future measurement of Hubble parameter and angular diameter distance through the full sky BAO survey and gave a encourage forecast that the relative error on $H(z)$ and d_A would be less than 1% at redshift $z > 0.5$. Therefore, with our method, the curvature parameter Ω_K can be constrained at a very high accuracy level in a model-independent way.

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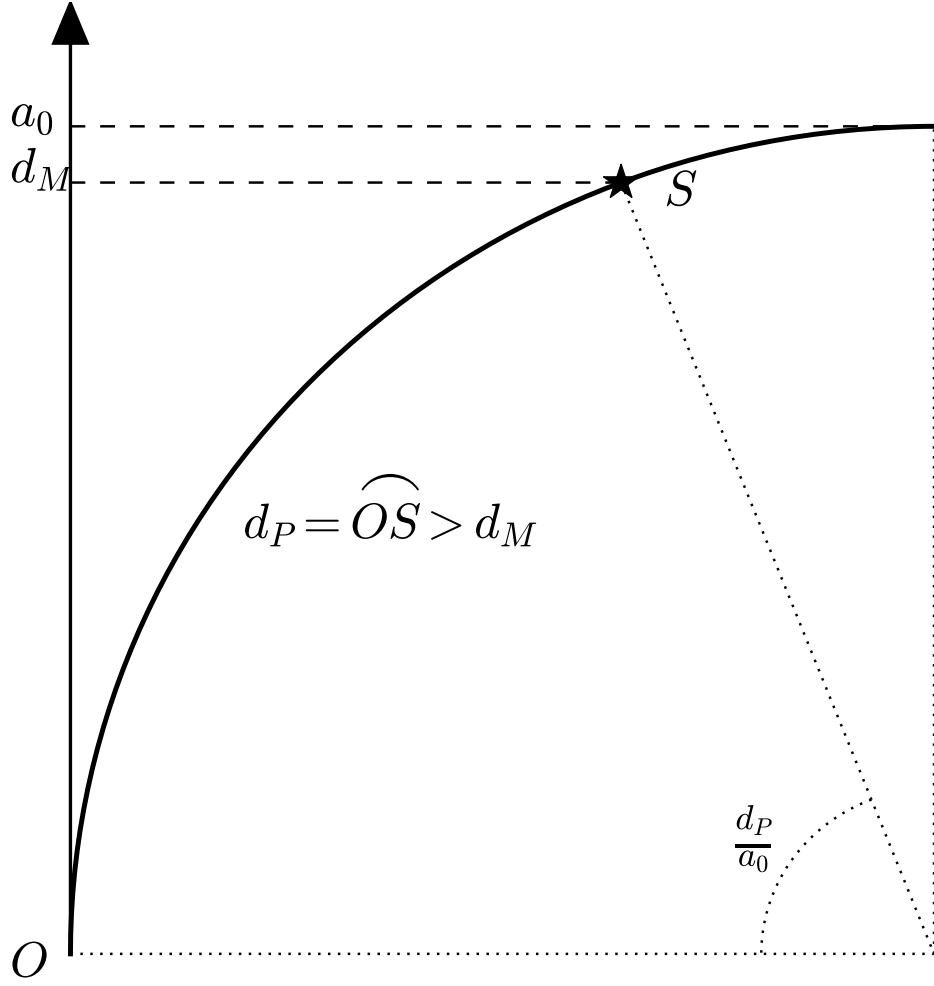


Fig. 1.— Illustration of the proper distance d_P and transverse comoving distance d_M in a closed universe. It is obvious that $d_M < d_P$.

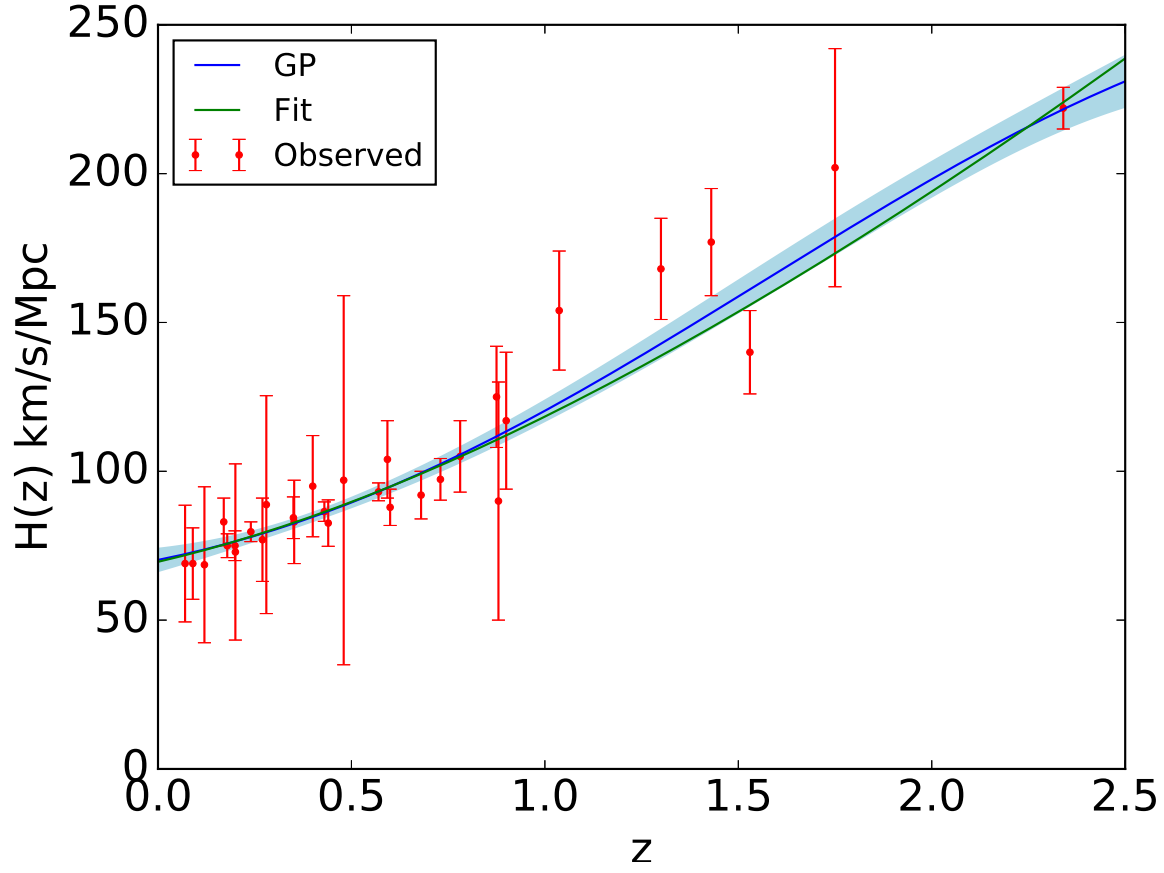


Fig. 2.— The blue curve and area show the $H(z)$ function and its 1σ confidence region reconstructed from GP method. The green curve shows the $H(z)$ function fitted with Λ CDM model. The red points and the error bars show the observed Hubble parameters and their 1σ errors.

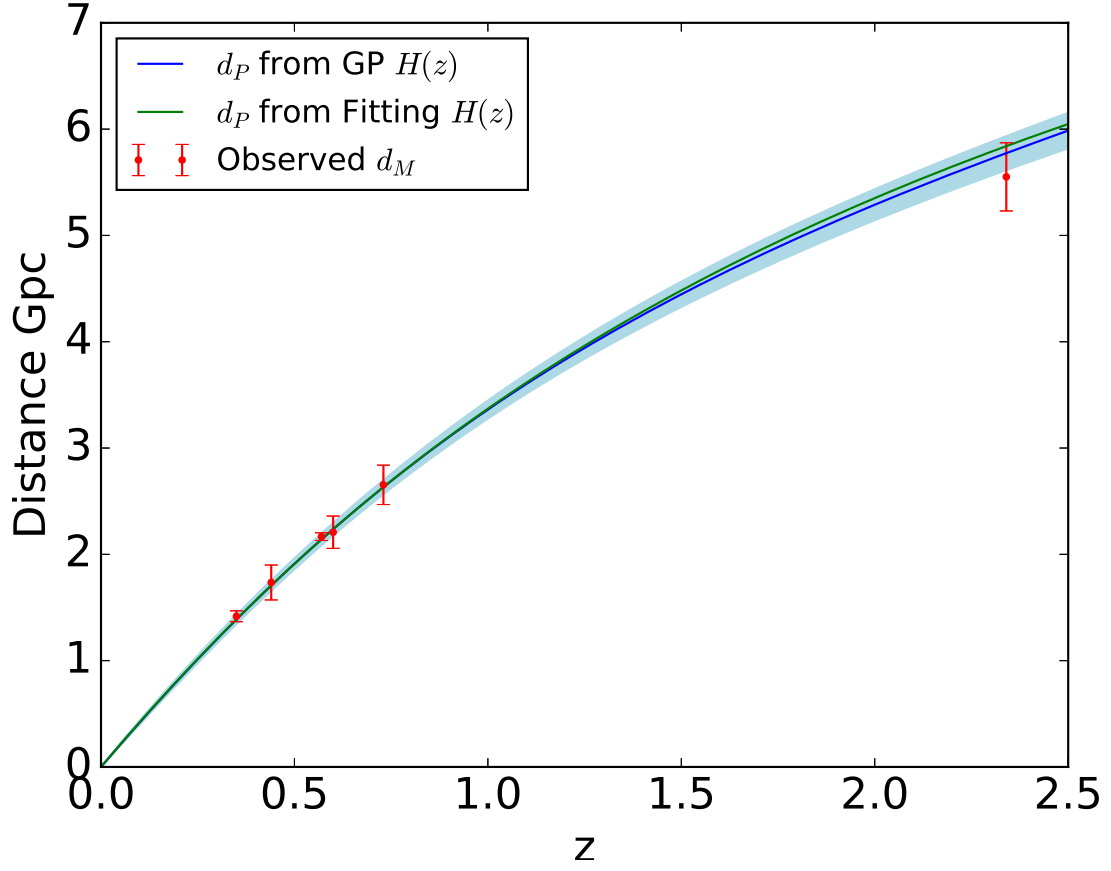


Fig. 3.— The blue curve and area show the $d_P(z)$ function and its 1σ confidence region derived from the GP- $H(z)$ function. The green curve shows the $d_P(z)$ function derived from the fitted $H(z)$ function. The red points and the error bars show the observed d_M and their 1σ errors.

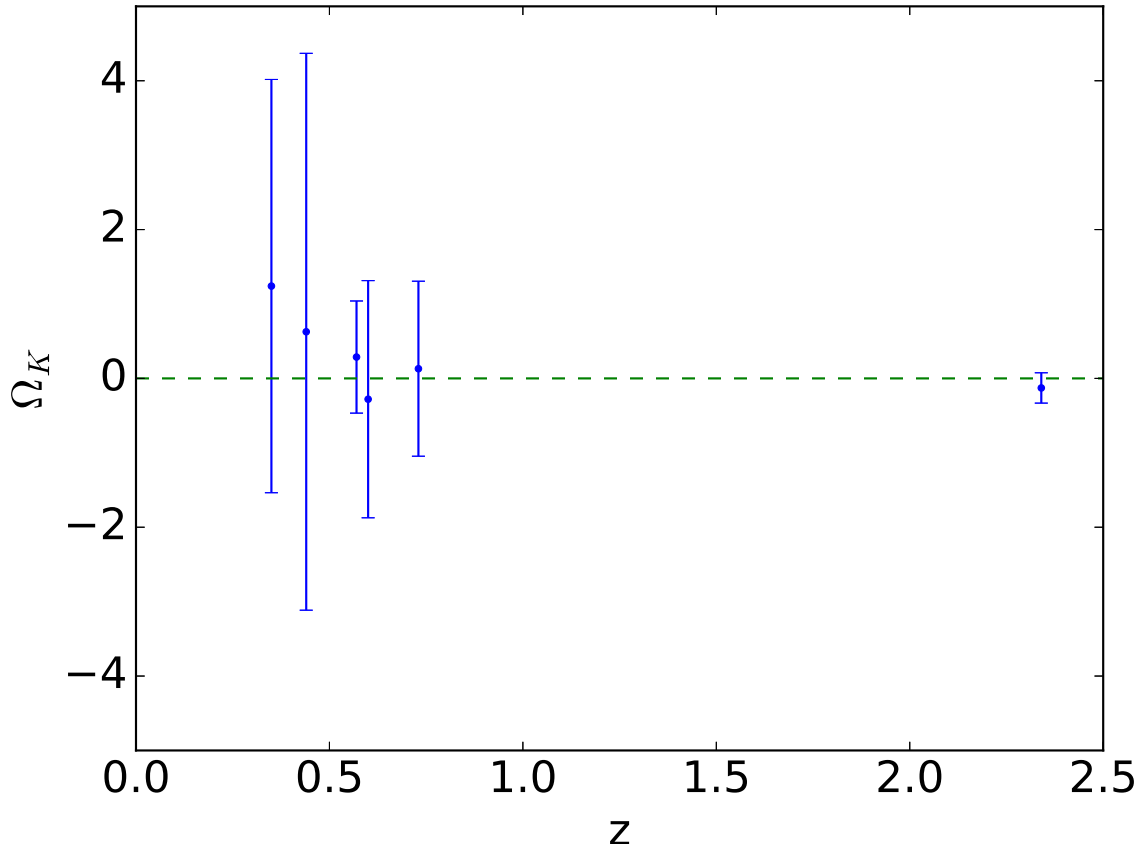


Fig. 4.— The Ω_K determined by comparing the GP- $d_P(z)$ function and observed d_M .

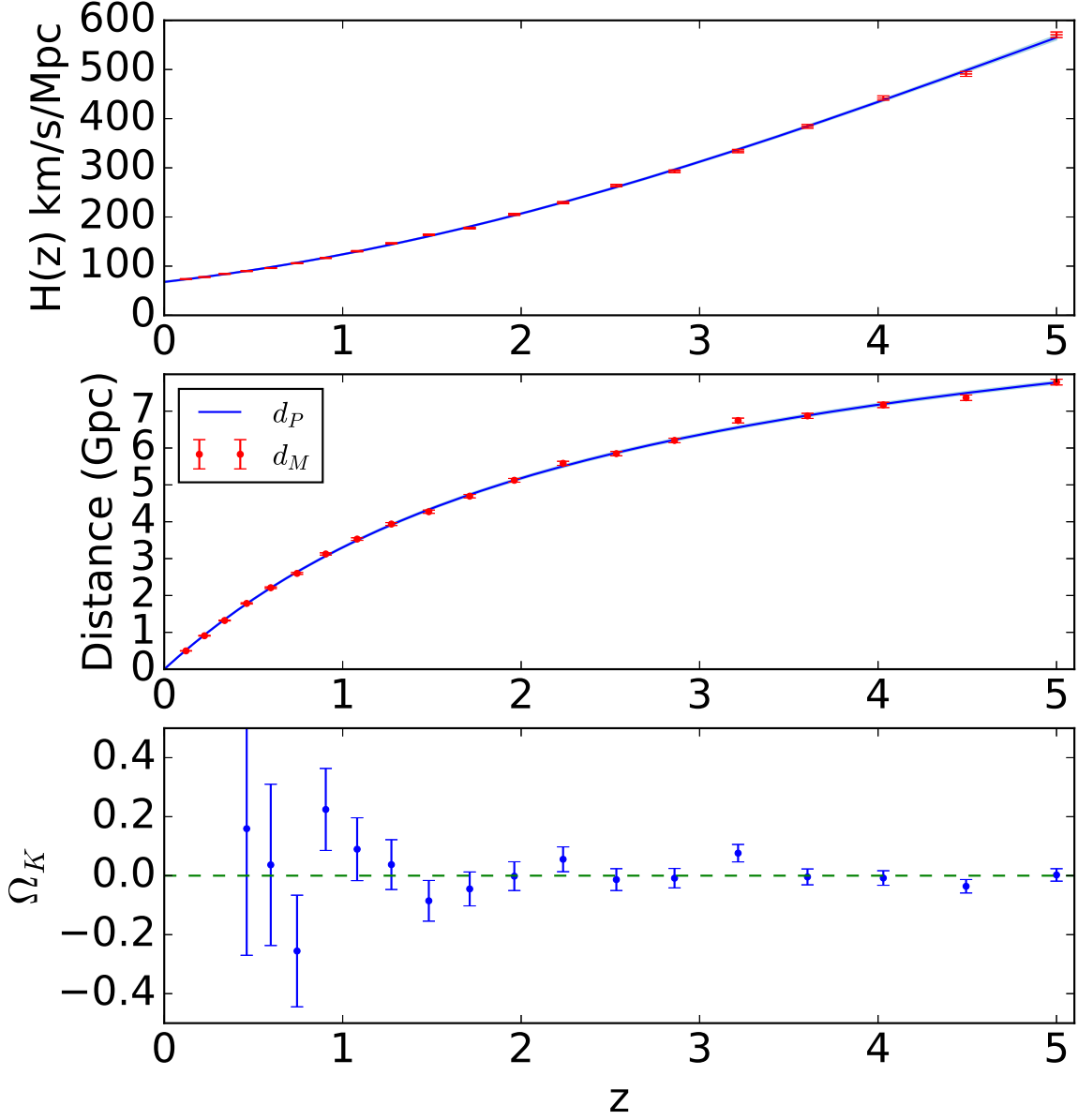


Fig. 5.— An example of the simulation for $\Omega_K = 0$ case. Top panel shows the mock Hubble parameter data and the GP- $H(z)$ function. Middle panel shows the mock d_M data and the GP- $d_P(z)$ function. The bottom panel shows the final Ω_K determined from these mock data.

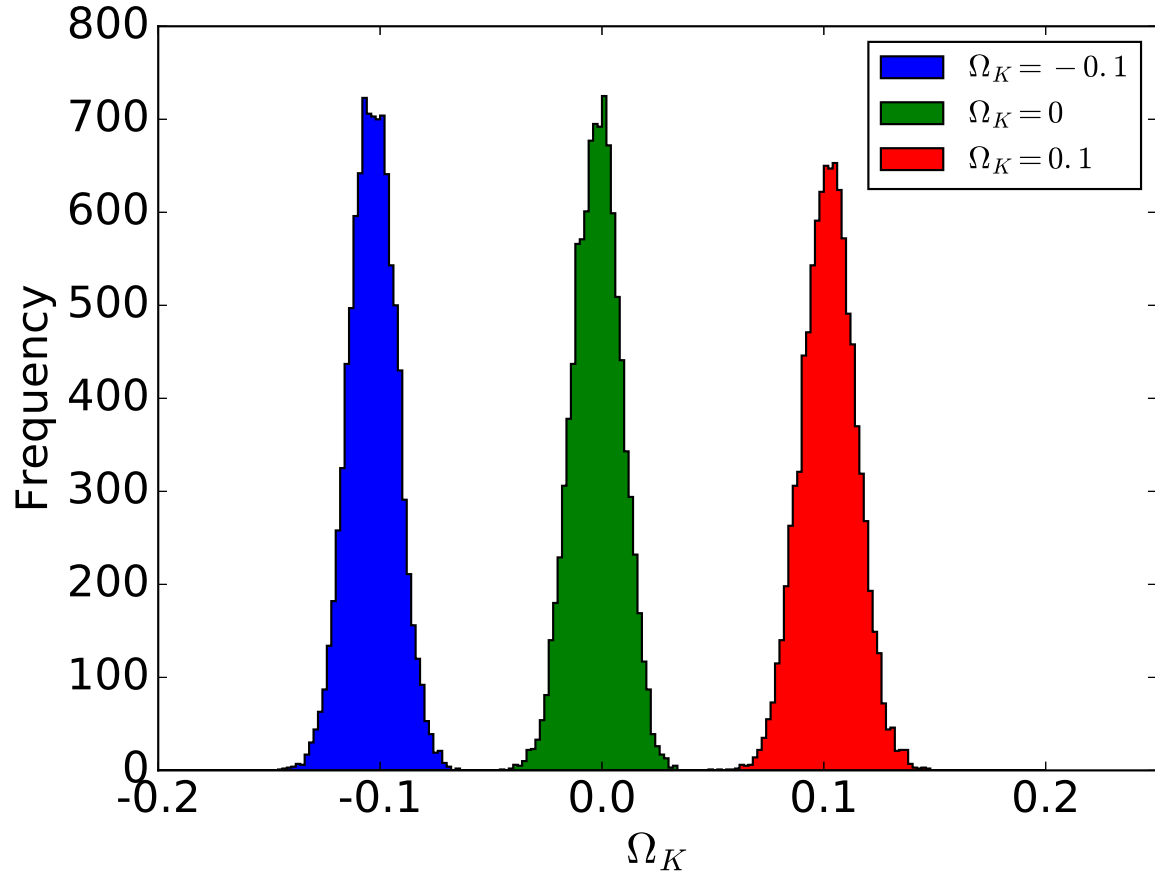


Fig. 6.— The distributions of Ω_K determined from simulated mock data based on background Λ CDM model with different prior curvatures using our method.

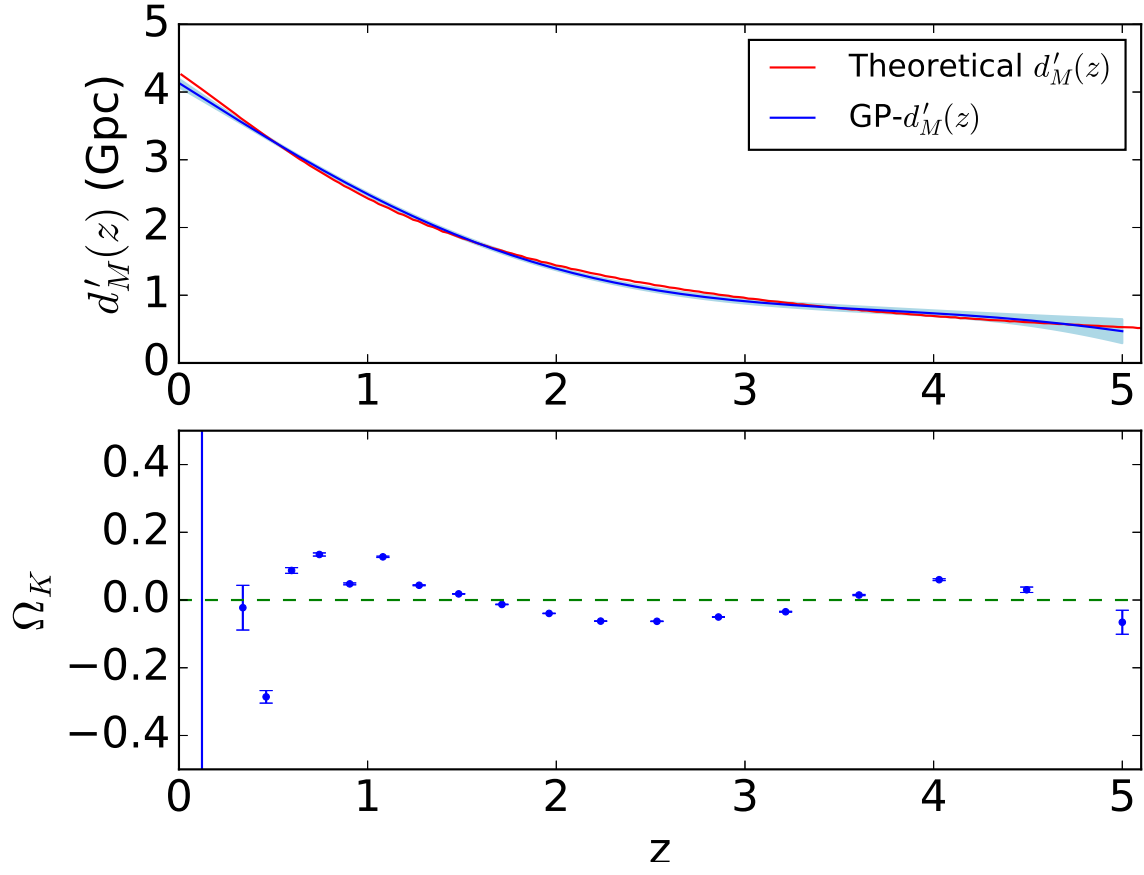


Fig. 7.— Same as Fig. 5 but for C07 method.

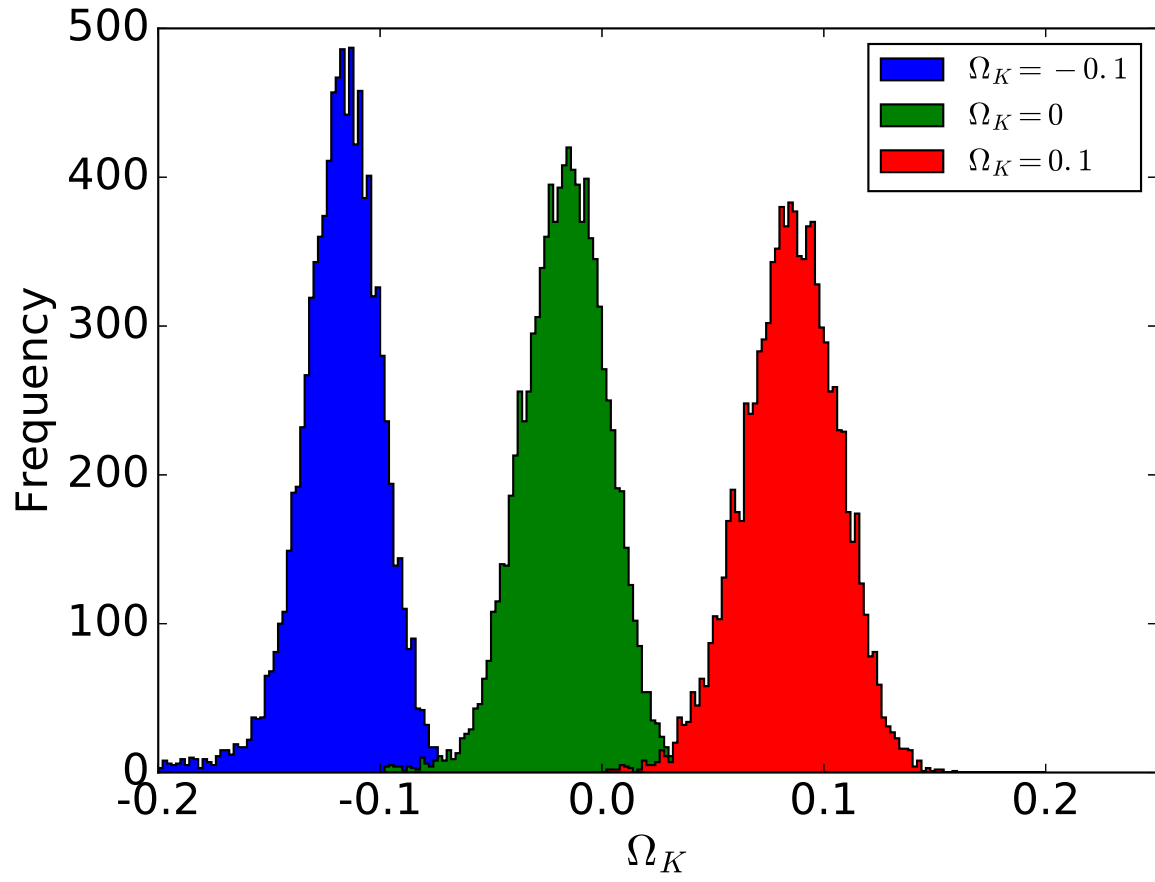


Fig. 8.— Same as Fig. 6 but for C07 method.

Table 1: The angular diameter distance $d_A(z)$ and their references.

z	$d_A(z)(\text{Mpc})$	Reference
0.44	1205 ± 114	
0.6	1380 ± 95	Blake et al. (2012)
0.73	1534 ± 107	
0.35	1050 ± 38	Xu et al. (2013)
0.57	1380 ± 23	Samushia et al. (2014)
2.34	1662 ± 96	Delubac et al. (2015)

Table 2: The data of Hubble parameter $H(z)$ and their references.

z	$H(z)$ km/s/kpc	reference
0.09	69 ± 12	Jimenez et al. (2003)
0.17	83 ± 8	
0.27	77 ± 14	
0.40	95 ± 17	
0.90	117 ± 23	
1.30	168 ± 17	
1.43	177 ± 18	
1.53	140 ± 14	
1.75	202 ± 40	
0.24	79.69 ± 3.32	Gaztañaga et al. (2009)
0.43	86.45 ± 3.27	
0.48	97 ± 62	Stern et al. (2010)
0.88	90 ± 40	
0.179	75 ± 4	Moresco et al. (2012)
0.199	75 ± 5	
0.352	83 ± 14	
0.593	104 ± 13	
0.680	92 ± 8	
0.781	105 ± 12	
0.875	125 ± 17	
1.037	154 ± 20	
0.44	82.6 ± 7.8	Blake et al. (2012)
0.60	87.9 ± 6.1	
0.73	97.3 ± 7.0	
0.35	84.4 ± 7.0	Xu et al. (2013)
0.07	69 ± 19.6	
0.12	68.6 ± 26.2	Zhang et al. (2014)
0.20	72.9 ± 29.6	
0.28	88.8 ± 36.6	
0.57	93.1 ± 3.0	
2.34	222 ± 7	Delubac et al. (2015)

Table 3: The Ω_K derived from equation (5) using our method.

z	0.35	0.44	0.57	0.60	0.73	2.34	Average
Ω_K	1.24 ± 2.78	0.63 ± 3.74	0.29 ± 0.75	-0.28 ± 1.59	0.13 ± 1.18	-0.13 ± 0.20	-0.09 ± 0.19